Published by Institute of Physics Publishing for SISSA
Received: *August 14, 2006*
Accepted: *October 3, 2006*
Published: *October 24, 2006*

Five-brane instantons vs. flux-induced gauging of isometries

Lilia Anguelova and Konstantinos Zoubos

Department of Physics, Queen Mary, University of London
Mile End Road, London E1 4NS, United Kingdom
E-mail: l.anguelova@qmul.ac.uk, k.zoubos@qmul.ac.uk

Abstract: In five-dimensional heterotic M-theory there is necessarily nonzero background flux, which leads to gauging of an isometry of the universal hypermultiplet moduli space. This isometry, however, is poised to be broken by M5-brane instanton effects. We show that, similarly to string theory, the background flux allows only brane instantons that preserve the above isometry. The zero-mode counting for the M5 instantons is related to the number of solutions of the Dirac equation on their worldvolume. We investigate that equation in the presence of generic background flux and also, in a particular case, with nonzero worldvolume flux.

Keywords: Flux compactifications, M-Theory.

JHEP10(2006)071

Contents

1. Introduction

Since the realization that nonzero background fluxes play an essential role for the resolution of the moduli stabilization problem in string and M- theory [1, 2], there has been an enormous amount of work on various flux compactifications. For a comprehensive review see [3]. The low-energy description of these compactifications is given by supergravity coupled to a certain number of matter and vector multiplets. The scalars belonging to these multiplets parametrize the moduli space of the four-dimensional effective theory. Depending on the preserved amount of supersymmetry, the presence of flux leads either to generation of a superpotential for (some of) the moduli fields or to gauging of some of the moduli space isometries.

These isometries survive perturbative corrections, but not necessarily non-perturbative ones. A source of the latter kind of contributions to the moduli space metric in $N = 2$ compactifications is provided by Euclidean branes.[1] Because of charge quantization, these brane instantons lead to the breaking (to a discrete subgroup) of certain continuous

[1] These nonperturbative corrections are essential for the stabilization of the Kähler moduli in type IIB compactifications [4].

JHEP10(2006)071

isometries. The latter are the shift symmetries implied by the gauge invariance of the 10d or 11d p-forms that couple to those branes. Since only continuous isometries can be gauged, there seems to be a potential clash between turning on background fluxes and taking into account brane-instanton effects. The resolution of this problem for string theory was addressed in [5]. It was shown there that, in the presence of D0- and D2-brane instantons, the background fluxes protect exactly the isometries that need to be gauged thus preserving the consistency of the supergravity description.

We will generalize their argument for the case of five-dimensional heterotic M-theory. A notable feature of the latter is that, unlike string theory, it does not admit vanishing background flux and so it always has a gauged isometry. This theory arises from considering Hořava-Witten on a CY(3) and is of interest for the following reason. The Hořava-Witten set-up is given by M-theory on an 11d manifold with boundaries or, equivalently, compactification of M-theory on an interval. To get to four dimensions, one further compactifies on a CY(3). This provides the strong coupling description of the $E_8 \times E_8$ heterotic string compactification on the same Calabi-Yau [6] and was argued to improve significantly on the phenomenological properties of the weakly coupled limit [7]. However, comparing to phenomenology leads to the conclusion that at some range of high energies the size of the interval is significantly larger than the size of the Calabi-Yau [8]. There is then an energy range or equivalently a time interval during the early universe evolution, when the universe is effectively five-dimensional.[2] The implications of this picture for cosmology have been studied extensively in recent years [9].

The effective action of 5d heterotic M-theory was obtained in [10, 11]. It is given by five-dimensional $N = 2$ gauged supergravity coupled to a certain number of vector- and hypermultiplets. Among them is a hypermultiplet that is the same for every Calabi-Yau and is hence called universal. The isometry that is gauged due to the presence of background flux is a symmetry of the universal hypermultiplet moduli space. On the other hand, one can show that this is precisely the symmetry that gets broken by the presence of M5-brane instantons wrapping the entire CY(3). (These instantons play a crucial role for moduli stabilization in 4d heterotic M-theory [12].) The reasoning is in exact parallel with the considerations of [13], which related 2- and 5-brane instantons to the breaking of particular isometries of the universal hypermultiplet target space in type IIA string theory on a CY(3). We will see that the apparent contradiction can be resolved along the lines of [5]. This leads to certain topological restrictions on the Calabi-Yau three-fold in order for M5 instantons to be present, but these restrictions can be eased by considering compactifications with non-standard embedding.

However, one could ask whether the M5 instantons contribute to the metric at all since a background flux can lift (some of) the fermionic zero modes living on the M5 worldvolume [14 − 16].[3] The latter works studied the influence of nonvanishing flux on the zero mode counting in the context of non-perturbatively generated superpotentials in $N = 1$ compactifications. Although it has been known for a while [18] that brane instantons

[2]At even earlier times (or higher energies) it is eleven-dimensional, whereas at later times (or lower energies) it becomes four-dimensional.

[3]This effect was first anticipated in [17].

can contribute to the superpotential, still many conceptual issues about the computation of their effects remain open. In the particular case of an M5-brane instanton wrapping a codimension-two cycle D of the internal space in an M-theory compactification on a CY(4), it was shown in [19], in the absence of flux, that the instanton gives a nonvanishing contribution only when the arithmetic genus $\chi(D, \mathcal{O}_D)$ of the cycle is equal to 1. This condition is required in order to cancel the U(1) anomaly related to rotations in the two internal dimensions that are normal to the M5 world-volume. In this compactification one obtains an exact result for the superpotential and this result can be translated into an exact superpotential for some particular cases in type IIB or heterotic compactifications via dualities.[4] The recent developments regarding zero-mode counting on brane-instanton world-volumes, that are based on the Dirac equation derived in [21], all address the issue of how the requirement $\chi(D, \mathcal{O}_D) = 1$ changes in the presence of background flux. However, even when the brane-instanton wraps the entire internal space, as is the case with the M5-instanton in 5d heterotic M-theory, and so there is no U(1) anomaly to be considered, still the supersymmetries that are broken by the brane-instanton generate fermionic zero modes in its world-volume theory. Hence studying the Dirac equation on the M5 world-volume can tell us when the brane-instanton can contribute to the moduli space metric.

As is well-known [7], the supersymmetric backgrounds in heterotic M-theory can have the following nonvanishing components of the 11d supergravity four-form G: $(2, 2, 0)$, $(2, 1, 1)$ and $(1, 2, 1)$, where the first two digits are the number of legs along the holomorphic and antiholomorphic indices of the Calabi-Yau three-fold respectively and the last one is along the interval direction. We will see that fluxes of type $(2, 2, 0)$ and $(2, 1, 1)$ do not affect the zero-mode counting on the M5 world-volume thus leading to four zero modes as in the fluxless case, whereas flux of type $(1, 2, 1)$ lifts all fermionic zero modes. We interpret this to mean that M5 instantons are incompatible with $(1, 2, 1)$ flux backgrounds. For anti-M5 instantons the situation is reversed, i.e. it is the $(2, 1, 1)$ type of flux that lifts all their zero modes.

Finally, we address the role of the self-dual three-form, living on the M5 brane, for the zero-mode counting of the world-volume fermions. This field has always been neglected in the literature because its presence complicates the Dirac equation quite a lot. However, it is a crucial ingredient in the generalization of the arguments of [5] to our case. So it is natural to ask how it would affect the above considerations. We do not undertake an investigation of the most general situation either, but for a particular case we are able to solve the Dirac equation for the most generic world-volume flux allowed by the M5 field equations. It turns out, that in this case the world-volume flux does not affect the zero mode counting.

The present paper is organized as follows. In section 2 we review necessary background material about 5d heterotic M-theory and its $(2, 2, 0)$-flux induced gauged isometry. In section 3 we summarize the results of [13] on the breaking of isometries of the universal hypermultiplet by 2- and 5-brane instantons and explain how this translates to 5d het-

[4] The relation with IIB comes via F-theory when the CY(4) is an elliptic fibration over a complex three-fold X. Then, if X is itself a $\mathbf{P}^1$ fibration over a two-fold Y, one can map to the heterotic string on a T^2 fibration over Y [20].

erotic M-theory. In section 4 we tackle the reconciliation of M5-brane instantons with the gauged isometry of heterotic M-theory. In 4.1 we show that the M5 instantons can indeed contribute to the moduli space metric on the basis of the zero mode counting on their world-volume. In 4.2 we argue that, similarly to the case considered in [5], the Gauss's law on the M5 world-volume forbids exactly the instantons that would have broken the gauged isometry. The existence of M5 instantons, which do not break this isometry, is related to a topological restriction on the internal CY(3). In 4.3 we show that this restriction can be eased in compactifications with non-standard embeddings. In section 5 we consider other types of background flux and show that the zero mode counting of the M5 world-volume Dirac equation is not affected by $(2,1,1)$ flux, whereas all zero modes are lifted by $(1,2,1)$ flux. In the appendix we show that the roles of these two types of flux are reversed for anti-M5 brane instantons. Finally, in section 6 we consider in a particular case the Dirac equation with nonvanishing world volume flux and find that the latter does not change the zero mode counting.

2. Gauged isometry

The effective five-dimensional theory arising from compactification of Hořava-Witten on a CY(3) was considered in [11]. It was shown there, that this is gauged supergravity coupled to $h^{1,1}-1$ vector multiplets and $h^{2,1}+1$ hypermultiplets. The $+1$ is the universal hypermultiplet that appears for any CY(3). Its bosonic field content is the following: the CY volume V, a real scalar σ that is dual to the external components of the 11d sugra 3-form C and a complex scalar ξ which comes from $C = \xi\Omega + \cdots$, where Ω is the holomorphic 3-form of the CY space. The presence of boundaries in eleven dimensions leads to a modification of the Bianchi identity for the field strength G of C:

$$dG = -\frac{1}{2\sqrt{2}\pi}\left(\frac{\kappa}{4\pi}\right)^{2/3}\sum_{a=1}^{2}\delta(x^{11}-x^{(a)})\left(\mathrm{tr}F^{(a)}\wedge F^{(a)} - \frac{1}{2}\mathrm{tr}R\wedge R\right), \tag{2.1}$$

where $x^{(1)} = 0$ and $x^{(2)} = \pi\rho$ are the positions of the two boundaries. As a result, only solutions with nonzero background flux are allowed. That is precisely the reason for the gauging of the effective 5d supergravity. This gauging will be important in the following. So, in order to explain how it occurs, let us first introduce the relevant notation and conventions of [11].

Let us start by taking the standard embedding of the spin connection in the first gauge group:[5]

$$\mathrm{tr}F^{(1)}\wedge F^{(1)} = \mathrm{tr}R\wedge R\,. \tag{2.2}$$

Then (2.1) becomes:

$$(dG)_{11ABCD} = -\frac{1}{4\sqrt{2}\pi}\left(\frac{\kappa}{4\pi}\right)^{2/3}\left[\delta(x^{11}) - \delta(x^{11}-\pi\rho)\right](\mathrm{tr}R\wedge R)_{ABCD}\,, \tag{2.3}$$

[5] The other E_8 gauge bundle is taken to be trivial.

where the indices A, B, C, D run over the six CY directions. Following [11], we introduce a basis ν^i, $i = 1, \dots, h^{2,2} = h^{1,1}$, of $(2,2)$-forms on the CY such that:

$$\frac{1}{v^{2/3}} \int_{C_i} \nu^j = \delta^j_i \,, \qquad \frac{1}{v} \int_X \nu^i \wedge \omega_j = \delta^i_j \,, \tag{2.4}$$

where C_i is a basis of 4-cycles, ω_j $-$ a basis of $(1,1)$ forms and v is a 6d reference volume. Now, one can expand the non-exact part of $\mathrm{tr} R \wedge R$ as:[6]

$$\mathrm{tr} R \wedge R|_{ne} = -8\sqrt{2}\pi \left(\frac{4\pi}{\kappa} \right)^{2/3} \alpha_i \nu^i \,. \tag{2.5}$$

The numerical coefficient above is chosen for convenience and

$$\alpha_i = \frac{\pi}{\sqrt{2}} \left(\frac{\kappa}{4\pi} \right)^{2/3} \frac{1}{v^{2/3}} \beta_i \,, \qquad \beta_i = -\frac{1}{8\pi^2} \int_{C_i} \mathrm{tr} R \wedge R \tag{2.6}$$

with β_i being integers related to the first Pontrjagin class of the CY. Using (2.5), the Bianchi identity (2.3) and the field equation, $D_I G^{IJKL} = 0$ with $I = 1, \dots, 11$, can be solved by the following background flux:

$$\begin{aligned} G_{ABCD} &= \alpha_i \nu^i_{ABCD} \epsilon(x^{11}) \,, \\ G_{ABC11} &= 0 \,, \end{aligned} \tag{2.7}$$

where $\epsilon(x^{11})$ is the step function defined to be $+1$ for $x^{11} > 0$ and -1 for $x^{11} < 0$.

Now we are ready to state the result of [11] about the flux-induced gauging of an isometry of the universal hypermultiplet moduli space. Let us denote the coordinates on the latter by $q^u \equiv (V, \sigma, \xi, \bar{\xi})^u$. Then the kinetic term of the universal hypermultiplet is [11]:

$$h_{uv} D_\alpha q^u D^\alpha q^v \,, \qquad D_\alpha q = (\partial_\alpha V, \, \partial_\alpha \sigma - 2\epsilon(x^{11}) \alpha_i \mathcal{A}^i_\alpha, \, \partial_\alpha \xi, \, \partial_\alpha \bar{\xi}) \,, \tag{2.8}$$

where h_{uv} is the metric on the quaternionic space $\mathrm{SU}(2,1)/\,\mathrm{U}(2)$ and $\mathcal{A}^i_\alpha$ are $h^{1,1}$ gauge fields arising via $C_{\alpha AB} = \frac{1}{6} \mathcal{A}^i_\alpha \omega_{iAB}$ with the index α running along the five non-CY dimensions[7]. Clearly, the isometry $\sigma \to \sigma + const$ of the metric h_{uv} is now gauged because of the nonzero background flux $G = \alpha_i \nu^i \, \epsilon(x^{11})$. The dualization that relates σ and $G_{\alpha\beta\gamma\delta}$ is accordingly modified:

$$G = \frac{1}{\sqrt{2}} V^{-2} *_5 \left[d\sigma - 2\epsilon(x^{11}) \alpha_i \mathcal{A}^i - i(\xi d\bar{\xi} - \bar{\xi} d\xi) \right] . \tag{2.9}$$

Finally, comparing (2.8) with the general expression for the extended derivative, $D_\alpha q^u = \partial_\alpha q^u + g \mathcal{A}^i_\alpha k^u_i$, we see that the Killing vectors k_i are:[8]

$$k_i = -2\epsilon(x^{11}) \alpha_i \partial_\sigma \,. \tag{2.10}$$

Although defining k_i as above will be of use for us, we should note that strictly speaking there is only one Killing vector: $k = \partial_\sigma$. So, in fact, the gauge field for the gauging is a linear combination of the graviphoton and the vectors from the vector multiplets, which is given by $\mathcal{A}_\alpha = -2\epsilon(x^{11}) \alpha_i \mathcal{A}^i_\alpha$.

[6] In the language of [11] this is denoted as $\mathrm{tr} R \wedge R|_0$ and referred to as 'zero mode part'. Its existence is exactly what leads to their $G_{ABCD}|_0 \neq 0$, which in modern terminology is really the background flux.

[7] The number of vector multiplets is $h^{1,1} - 1$ because one of the $h^{1,1}$ vectors $\mathcal{A}^i$ (rather, a certain combination of them) is the graviphoton of the supergravity multiplet.

[8] For convenience we set the gauge coupling constant $g = 1$; or, equivalently, we absorb it in the definition of α_i.

3. Five-brane instantons

In [13] it was shown that 5-brane and 2-brane instantons lead to the breaking of certain isometries of the universal hypermultiplet moduli space. The considerations of that paper were in the context of type IIA compactifications on a CY(3) to a four-dimensional effective theory with $N = 2$ supersymmetry. In this case the bosonic content of the universal hypermultiplet is made up of the dilaton φ, a real scalar D that is dual to the external components of the B-field and a complex scalar C originating from $A_{(3)} = C\Omega$, where $A_{(3)}$ is the RR 3-form potential. The precise relation between D and H, which is locally $H_{\mu\nu\rho} = (dB)_{\mu\nu\rho}$ with μ, ν, ρ being four-dimensional indices, is given by

$$H = e^{4\varphi} *_4 \left[2dD + i(\bar{C}dC - Cd\bar{C})\right] . \tag{3.1}$$

The manifold parametrized by $\varphi, D, C, \bar{C}$ is the coset $\mathrm{SU}(2,1)/\,\mathrm{U}(2)$ [22]. In terms of the complex coordinates $S, \bar{S}, C, \bar{C}$, where

$$S \equiv e^{-2\varphi} + 2iD + C\bar{C} \,, \tag{3.2}$$

this coset has the following symmetries:

$$\begin{aligned} S &\rightarrow S + i\alpha + 2(\gamma + i\beta)C + \gamma^2 + \beta^2 \\ C &\rightarrow C + \gamma - i\beta \,, \end{aligned} \tag{3.3}$$

which correspond to constant shifts of the NS axion D and the RR scalars $C, \bar{C}$. These symmetries are invariances of the classical Lagrangian. Their existence is implied by the gauge transformations[9] of the 3-form H and 4-form $F_{(4)} = dA_{(3)}$. As shown in [23], they survive when sigma-model perturbative corrections are taken into account. They are also expected to survive in string perturbation theory [24].[10] (By contrast, the remaining symmetries of the coset $\mathrm{SU}(2,1)/\,\mathrm{U}(2)$ are generically broken by perturbative effects.[11]) However, non-perturbative corrections due to membrane and five-brane instantons will break the isometries in (3.3) [13]. Let us recall the argument for this.

It was shown in [13] that the symmetries (3.3) give rise to the Noëther currents

$$\begin{aligned} J_\alpha &= \frac{i}{\kappa_4^2} e^{2K} \left(dS - d\bar{S} + 2(Cd\bar{C} - 2\bar{C}dC)\right) \,, \\ J_\beta &= -\frac{2i}{\kappa_4^2} e^{K} (dC - d\bar{C}) + 2(C + \bar{C})J_\alpha \,, \\ J_\gamma &= -\frac{2}{\kappa_4^2} e^{K} (dC + d\bar{C}) - 2i(C - \bar{C})J_\alpha \,, \end{aligned} \tag{3.4}$$

[9]Recall that gauge invariance implies lack of non-derivative couplings of the corresponding potentials, which in turn gives rise to exactly the shift symmetries of the scalars obtained from reduction of those potentials.

[10]The perturbative corrections to the moduli space of the universal hypermultiplet were first addressed in [24] and studied more thoroughly in [25, 26].

[11]For more details on the symmetries of the coset $\mathrm{SU}(2,1)/\,\mathrm{U}(2)$ see [27].

JHEP10(2006)071

where K is the Kähler potential $K = -\ln(S + \bar{S} - 2C\bar{C})$. Integrating these currents over a three-cycle Σ_3 in the 4d external space, one obtains the corresponding conserved charges:

$$Q_{\alpha,\beta,\gamma} = \int_{\Sigma_3} *_4 J_{\alpha,\beta,\gamma} \,. \tag{3.5}$$

However, these charges can be shown to be related to the presence of 5-brane (Q_α) and 2-brane (Q_β, Q_γ) instantons. For example for the 5-brane, the case that will be of importance for us, one can easily see from (3.2) and (3.1) that

$$Q_\alpha = \int_{\Sigma_3} *_4 J_\alpha = \int_{\Sigma_3} H \,, \tag{3.6}$$

where we also used that $K = -\ln(S + \bar{S} - 2C\bar{C}) = 2\varphi$. Clearly then, Q_α is the five-brane charge and so charge quantization implies that the presence of 5-brane instantons breaks the symmetry generated by J_α (i.e. the symmetry $S \to S + i\alpha$) to a discrete subgroup.[12]

Let us now compare the above type IIA compactification to a 4d $N = 2$ theory with the compactification of Hořava-Witten on CY(3), that leads to a five-dimensional effective theory. In both cases there are eight preserved supercharges. In addition, the scalars $V, \sigma, \xi, \bar{\xi}$, introduced in the previous section, parametrize the same quaternionic manifold, $\mathrm{SU}(2,1)/\,\mathrm{U}(2)$, as do $\varphi, D, C, \bar{C}$. The coordinate transformation between the two sets of coordinates is:

$$\sigma = 2D \,, \qquad V = e^{-2\varphi} \,, \qquad \xi = C \,. \tag{3.7}$$

Hence, the same symmetries as (3.3) are also present for the moduli space of the universal hypermultiplet in the 5d theory. And similarly to the type IIA case this leads, upon using (2.9), to the shift symmetry $\sigma \to \sigma + \alpha$ being broken by the presence of five-brane charge[13]

$$Q_\alpha = \int_{\Sigma_4} *_5 J_\alpha = \int_{\Sigma_4} G \,, \tag{3.8}$$

where Σ_4 is a four-cycle in the five-dimensional external space.[14]

However, as we recalled in section 2, the isometry $\sigma \to \sigma + const$ is gauged because of the presence of background flux in the CY compactification of Hořava-Witten theory. Since only continuous isometries can be gauged, it appears therefore that there is a clash between this gauging and the possible five-brane instanton effects.

4. Flux-induced gauging vs M5 instantons

In the present section we address the reconciliation of the above competing effects. First, in 4.1 we explain that five-brane instantons can exist in the theory we are considering and so there is indeed a potential problem. In 4.2 we show that the latter is resolved,

[12] For further study of 2- and 5-brane instanton effects on the universal hypermultiplet moduli space in the supergravity description see [28].

[13] Since $D = ImS$, clearly the shift $S \to S + i\alpha$ is actually $D \to D + \alpha$.

[14] A similar conclusion was reached in [29] from a different point of view.

JHEP10(2006)071

similarly to the string theory case, by requiring that Gauss' law is obeyed on the brane-instanton world-volume. This implies that the background flux allows only instantons that would not break the gauged isometry. If such M5's are to exist, then the CY has to satisfy some topological constraints. In 4.3 we show that these constraints can be made less restrictive by considering compactifications with non-standard embedding due to the presence of Minkowski M5-branes in the bulk.

4.1 Fermionic zero modes and 5-brane instantons

To claim that there is a possible clash between the gauged isometry, parametrized by the coordinate σ, and five-brane instantons, let us first convince ourselves that the latter are not forbidden by supersymmetry. In [30] it was shown that, to first order in the $\kappa^{2/3}$ expansion of Hořava-Witten theory, supersymmetry allows only Minkowski membranes that stretch between the two boundaries and Minkowski five-branes that are parallel to the boundaries. These are solutions in which the M2 and M5 branes are part of the background. Nevertheless, it is natural to expect that the same conclusion will hold for their instantonic counterparts. Indeed, it was shown in [31] that the only M2 instantons, which are compatible with supersymmetry, are given by membranes wrapping holomorphic curves on the boundaries and stretching along the eleventh direction. The argument was based on analyzing what embeddings of the membrane worldvolume into the eleven-dimensional spacetime allow solutions of $\Gamma\epsilon = \epsilon$, where Γ is the worldvolume operator that defines the κ-symmetry transformation and ϵ is the supersymmetry parameter. Clearly, one can perform an analogous computation for the 5-brane instantons. However, it will be of future use for us to verify the existence of instantons, due to M5-branes wrapping the whole CY(3), by counting the fermionic zero-modes on the brane worldvolume.

Recall that the supersymmetries that are broken by the presence of a brane generate fermionic zero modes on its worldvolume.[15] In order for a brane-instanton to be able to contribute to the moduli space metric of the external theory, the Dirac equation for its worldvolume fermions has to have four zero modes. Note that, in addition to the zero modes coming from the broken supercharges, there can also be zero modes related to internal degrees of freedom (i.e., superpartners of bosonic deformations of the internal cycle). Furthermore, recently it was shown that background fluxes can change the zero-mode counting significantly [14, 15].[16] These results were based on the Dirac equation for the M5-worldvolume fermions in a nonvanishing background, derived in [21]. The latter work considers only the quadratic terms in the fermionic worldvolume action. However, this is enough for ruling out M5-brane instanton contributions (in the case of less than four zero modes) since the higher (interaction) terms can only lift zero modes of the quadratic action but not introduce new ones.[17] As we reviewed in section 2, in the case of interest for

[15]For a nice recent discussion of this point see [16].

[16]An apparent contradiction in this kind of analysis, related to the consistent inclusion of non-perturbative effects in the minimization of the 4d effective superpotential, was resolved in [32].

[17]Recall that, in principle, counting arguments can only be enough for ruling out certain contributions. However, they do not necessarily imply a non-vanishing correction since even when they do not rule it out, the explicit computation may end up giving zero.

JHEP10(2006)071

us there is nonvanishing background flux. So we are going to show that there are exactly four zero modes on the worldvolume of an M5 wrapping a CY 3-fold by specializing the Dirac equation of [21] to our set-up.

Let us start by decomposing the eleven-dimensional spinor in the appropriate way. To begin with, it transforms in the **32** of $\mathrm{SO}(1,10)$, or in $\mathrm{Spin}(1,10)$ to be more precise. Compactifying on $CY \times M_4 \times S^1/\mathbb{Z}_2$, the group $\mathrm{SO}(1,10)$ gets broken to $\mathrm{SU}(3) \times \mathrm{SO}(1,3)$. After analytic continuation to Euclidean space, the latter group becomes $\mathrm{SU}(3) \times \mathrm{SO}(4)$. Hence the spinor, θ, on the worldvolume of an M5-brane wrapping the CY(3) transforms in the $(\mathbf{1},\mathbf{4}) \oplus (\mathbf{3},\mathbf{4}) \oplus (\bar{\mathbf{3}},\mathbf{4}) \oplus (\bar{\mathbf{1}},\mathbf{4})$. Defining the Clifford vacuum $|\Omega\rangle$ by

$$\gamma^a |\Omega\rangle = 0\,, \tag{4.1}$$

where the index a runs over the holomorphic coordinates of the CY, one can expand θ as

$$\theta = \phi|\Omega\rangle + \ \phi_{\bar{a}\bar{b}}\gamma^{\bar{a}\bar{b}}|\Omega\rangle\,. \tag{4.2}$$

Recall that this expansion contains only terms with even number of indices because after κ-symmetry gauge-fixing one is left with a chiral fermion on the worldvolume [21]. Also, we have suppressed the **4** index on ϕ, $\phi_{\bar{a}\bar{b}}$ for simplicity.

Let us now turn to the Dirac equation [21]:

$$\gamma_A m^{AB}\nabla_B\theta + \frac{1}{24}\left[\gamma^{\alpha\beta\delta}\gamma^A(2\delta_A^B - m_A{}^B)G_{B\alpha\beta\delta} + \gamma^\alpha\gamma^{ABC}(2\delta_A^D - 3m_A{}^D)G_{DBC\alpha}\right]\theta = 0\,. \tag{4.3}$$

As before, indices α, β, δ run over the five dimensions that are transverse to the CY (and so to the M5-brane instanton) and A, B, C, D run along the six worldvolume directions. Also, the matrix m is determined by the worldvolume 3-form flux h via

$$m_A{}^B = \delta_A{}^B - 2h_{ACD}h^{BCD}\,. \tag{4.4}$$

For convenience, from now on we will absorb the $1/24$ factor in the definition of the background flux G. To simplify the problem we will consider in the following, as in all existing literature, only vanishing worldvolume flux. (We will have more to say about the $h \neq 0$ case in section 6.) Hence (4.3) reduces to:

$$\gamma^a\nabla_a\theta + \gamma^{\bar{a}}\nabla_{\bar{a}}\theta + \left(\gamma^{\alpha\beta\delta}\gamma^A G_{A\alpha\beta\delta} - \gamma^\alpha\gamma^{ABC}G_{ABC\alpha}\right)\theta = 0\,. \tag{4.5}$$

Since the background flux in (2.7) has only G_{ABCD} nonzero components, clearly the Dirac equation (4.5) is completely unaffected by the flux. Hence the counting of zero modes gives four, which is what is necessary for the M5 instanton to contribute to the metric. Indeed, substituting (4.2) in (4.5), one finds

$$\begin{aligned}\partial_{[\bar{a}}\phi_{\bar{b}\bar{c}]}\gamma^{\bar{a}\bar{b}\bar{c}}|\Omega\rangle &= 0\\ (\partial_{\bar{a}}\phi + 4g^{\bar{b}c}\partial_c\phi_{\bar{b}\bar{a}})\gamma^{\bar{a}}|\Omega\rangle &= 0\,,\end{aligned} \tag{4.6}$$

JHEP10(2006)071

where $g_{a\bar{b}}$ is the Kähler metric on the CY. Hence, as in [14], the forms ϕ and $\phi_{\bar{a}\bar{b}}$ are harmonic.[18] However, as $h^{0,2} = 0$ for a CY(3), it follows that $\phi_{\bar{a}\bar{b}} = 0$. So we are left with the single component ϕ, which due to the suppressed index in the **4** of SO(4) means that there are exactly four zero modes. Therefore, M5-brane instantons are in principle allowed in the theory under consideration, despite the existence of the flux-induced gauging of the shift symmetry along σ (which is the isometry they are supposed to break).

4.2 Reconciling M5 instantons with flux-induced isometry gauging

It turns out that the resolution of the above puzzle is along the lines of [5], which considered D2-brane instantons and flux-induced gauging of isometries in type II strings. The idea is the following. The correction to the moduli space metric, due to brane-instanton effects, is of the form $Te^{S_{inst}}$, where S_{inst} is the brane action and the prefactor T is made up of one-loop determinants. Generically T can depend on some of the moduli but not on the p-form ones, whose shift symmetries are broken by the brane-instanton (in our case, the coordinate σ), because the dependence on the latter is fixed by the charge of the instanton.[19] In other words, the above p-form moduli enter the brane-instanton induced correction only via S_{inst}. Hence, it is enough to show that the change of S_{inst}, generated by the Killing vector of the isometry to be gauged, vanishes for brane-instantons that are compatible with the background flux (i.e., satisfy the appropriate Gauss' law).

Let us start by recalling the covariant Minkowski M5-brane worldvolume action [33]:

$$\begin{aligned} S_{M5} = &- \int d^6x \left(\sqrt{-det(g_{mn} + i\tilde{H}_{mn})} - \frac{\sqrt{-g}}{4\partial_q a \partial^q a} \partial_l a \, H^{*lmn} H_{mnp} \, \partial^p a \right) \\ &- \int \left(C^{(6)} + \frac{1}{2} F \wedge C^{(3)} \right) , \end{aligned} \tag{4.7}$$

where m, n are worldvolume indices, $a(x)$ is an auxiliary field and

$$H_{lmn} = F_{lmn} - C^{(3)}_{lmn} \, , \qquad H^{*lmn} = \frac{1}{3!\sqrt{-g}} \epsilon^{lmnpqr} H_{pqr} \, , \qquad \tilde{H}_{mn} = \frac{H^*_{mnl} \partial^l a}{\sqrt{(\partial a)^2}} \, . \tag{4.8}$$

Finally, $F = dA$ is the field-strength of the 5-brane worldvolume two-form field A. Recall that F (or, equivalently, H) satisfies a non-linear self-duality condition and there is a non-linear field redefinition that relates it to a worldvolume 3-form h, which obeys an ordinary linear self-duality constraint but is not related to a potential.[20] Let us also note that the

[18] Recall that this can be derived in the following way. Acting with $\nabla^{\bar{a}}$ on the second equation in (4.6), we obtain that $\Delta\phi = 0$. On the other hand, acting with $\nabla_{\bar{d}}$ on the second line of (4.6) and anti-symmetrizing w.r.t. the pair $(\bar{a}, \bar{d})$ gives, after adding the result of the action of $\nabla^{\bar{a}}$ on the first line of (4.6), that $\Delta\phi_{\bar{b}\bar{c}} = 0$. These manipulations use the fact that on a Kähler manifold the only nonvanishing components of the Christoffel symbols are Γ^c_{ab} and $\Gamma^{\bar{c}}_{\bar{a}\bar{b}}$ and also $R_{a\bar{b}c\bar{d}} = R_{a\bar{d}c\bar{b}} = R_{c\bar{b}a\bar{d}}$.

[19] In the case of brane-instanton generated superpotentials (i.e., in $N = 1$ compactifications), T is a function of the complex structure moduli but, due to holomorphy, not of the Kähler ones (see [19]). However, for non-perturbatively generated corrections to the moduli space metric, clearly there is no holomorphy and so one cannot exclude Kähler moduli dependence of the instanton prefactor.

[20] More precisely, the relation between the two fields is $H_{\underline{lmn}} = (\delta^{\underline{r}}_{\underline{l}} - 2h_{\underline{lpq}} h^{\underline{rpq}})(\delta^{\underline{s}}_{\underline{m}} - 2h_{\underline{mp'q'}} h^{\underline{sp'q'}}) h_{\underline{nrs}}$ [34] in terms of flat indices, or equivalently $H_{lmn} = (m^{-1})_l{}^p h_{mnp}$ [35].

auxiliary field $a(x)$ can be gauged away [33] and in the gauge, in which $a(x)$ is equal to one of the worldvolume coordinates, the second term on the first line of (4.7) is of the form

$$\int (F - C^{(3)}) \wedge (F - C^{(3)}) \,. \tag{4.9}$$

Euclidean continuation of the above action is achieved, as usual, by taking the worldvolume time x^0 to $\pm i x^0$.

Now, in order to follow the logic of [5] we want to see what is the explicit dependence of the five-brane action on the scalar σ so that we can compute the change of S_{M5} under the transformation generated by the vector field k_i (see eq. (2.10)). Since $*_5 dC^{(3)} = d\sigma$ with $C^{(3)}$ having only external (which in particular means non-worldvolume) indices, no terms with $C^{(3)}$ in S_{M5} contribute σ-dependence.[21] On the other hand, the 11d duality $*_{11} dC^{(3)} = dC^{(6)}$ descends to $C^{(6)} = \sigma w$, where w is the CY volume form. Hence

$$\delta S_{M5} = k_i(S_{M5}) = k_i(\int C^{(6)}) = -2\epsilon(x^{11})\alpha_i \,. \tag{4.10}$$

So, as long as $\alpha_i \neq 0$, the action is not invariant. However, on the M5 worldvolume, X, $dH = -\frac{1}{4}G$ [34], where G is the (pullback of the) background flux. Therefore, on X the flux G has to be cohomologically trivial. From (2.7) this implies that on X

$$\alpha_i = 0 \qquad \forall i \,, \tag{4.11}$$

which restores the invariance of S_{M5}. So the background flux does not allow five-brane instantons, unless the CY(3) is such that

$$\int_{C_i} \mathrm{tr} R \wedge R = 0 \tag{4.12}$$

for every 4-cycle C_i, in which case the isometry $\sigma \to \sigma + \mathit{const}$ is not gauged anyway. Clearly, the conditions (4.12) are satisfied for Calabi-Yau's with vanishing first Pontrjagin class.

4.3 Non-standard embedding

In (2.2) we assumed the standard relation between the E_8 gauge group of the visible boundary and the spin connection of the CY space. However, non-standard embeddings allow other (than E_6) unbroken gauge groups on the visible boundary and so have attracted a lot of phenomenological interest on their own. (They were introduced in the context of the weakly coupled heterotic string back in [36].) Even richer possibilities for the breaking of $E_8 \times E_8$ arise when one considers M5-branes parallel to the boundaries and situated at various positions along the interval. These five-branes are extending along the four external directions and wrapping a holomorphic curve in the CY(3). Including them is incompatible

[21] We use $*_5 dC^{(3)} = d\sigma$ instead of the full relation (2.9), because we concentrate only on the σ (as opposed to ξ) dependence and keep only terms linear in $\kappa^{2/3}$. (As the Killing vector is proportional to $\alpha_i \sim \kappa^{2/3}$, in $k_i(S_{M5})$ the terms of $\mathcal{O}(\kappa^{2/3})$ come from the part of S_{M5} that is zeroth order in $\kappa^{2/3}$.)

with the standard embedding. For an undoubtedly incomplete list of the vast literature on phenomenology of these compactifications, see [37].

The low-energy effective theory of the strongly coupled heterotic $E_8 \times E_8$ string on a CY(3) with non-standard embedding (with or without five-branes) was derived in [38]. Depending on the energy regime of interest it is useful to compactify either to four or to five dimensions. In the latter case, one again obtains five-dimensional gauged supergravity in the bulk. The effective theory has the same form as the one for the standard embedding [11], but the gauging parameters α_i are now different from (2.6). For non-standard embedding without five-branes:[22]

$$\alpha_i \sim \int_{C_i} \left(\mathrm{tr} F^{(1)} \wedge F^{(1)} - \frac{1}{2} \mathrm{tr} R \wedge R \right) , \tag{4.13}$$

whereas in the presence of n M5-branes, positioned at $x_1, \ldots, x_n$ along the eleventh dimension, the parameters α_i change in each interval $x_k \leq x^{11} \leq x_{k+1}$. More precisely, one finds [38]:

$$\alpha_i^{(k)} \sim \sum_{m=0}^{k} \beta_i^{(k)} \epsilon(x^{11}) \qquad \text{for} \qquad x^{11} \in (x_k, x_{k+1}) \, , \tag{4.14}$$

where x_0 and x_{n+1} denote the positions of the visible and hidden boundaries respectively and the integers $\beta_i^{(k)} = \int_{C_i} J^{(k)}$ are topological invariants giving the intersection number of the k-th five-brane with the four-cycle C_i for $k = 1, \ldots, n$ and $i = 1, \ldots, h^{2,2}$.

As in section 2, the isometry of the universal hypermultiplet moduli space that is gauged is generated by $k = \partial_\sigma$. So, following the arguments of section 4.2, we again conclude that five-brane instantons are allowed only when the relevant gauging parameters $\alpha_i^{(k)}$ vanish. However, since the Minkowski M5-branes are themselves magnetic sources of flux, one can achieve the vanishing of $\alpha_i^{(k)}$ with an appropriate choice of M5-branes in the bulk without the need to impose (4.12) on the CY(3). Hence, the topological conditions that the Calabi-Yau should satisfy, so that there can be 5-brane instantons, are least restrictive for non-standard-embedding vacua with bulk M5-branes.

5. More general background flux

So far we have considered only background flux of type $(2,2,0)$, i.e. $G_{a\bar{b}c\bar{d}}$. As we saw, it does not affect the Dirac equation of the M5 world-volume fermions. However, in heterotic M-theory one could also have supersymmetric backgrounds with nonvanishing flux components of type $(2,1,1)$ and $(1,2,1)$, i.e. $G_{ab\bar{c}11}$ and $G_{\bar{a}\bar{b}c11}$; see, for example, [39]. Such components appear also in a background including the gauge five-brane considered in [30].[23] Let us now see how they modify the zero-mode counting for M5-brane instantons.

[22]The precise numerical coefficients will not be important for us, so we will omit them for clarity.

[23]This is the lift to strong coupling of the heterotic string solution of [40].

In a vacuum with nonzero $G_{ab\bar{c}11}$ and $G_{\bar{a}\bar{b}c11}$, the Dirac equation on the worldvolume of an M5 instanton[24] (still neglecting the worldvolume flux h) acquires the form:

$$\begin{aligned}\left(\partial_{[\bar{a}}\phi_{\bar{b}\bar{c}]} + 4G_{[\bar{a}\bar{b}}{}^{\bar{d}}{}_{|11|}\phi_{\bar{c}]\bar{d}} + 2G^{\bar{d}}{}_{\bar{d}\,[\bar{a}\,|11|}\phi_{\bar{b}\bar{c}]}\right)\gamma^{\bar{a}\bar{b}\bar{c}}|\Omega\rangle &= 0 \\ \left(\partial_{\bar{a}}\phi + 4g^{\bar{b}c}\partial_c\phi_{\bar{b}\bar{a}} - 8G_{\bar{a}11bc}\phi^{bc} + 8G_{c11}{}^{c\bar{b}}\phi_{\bar{b}\bar{a}} + 2G_{\bar{a}\bar{c}}{}^{\bar{c}}{}_{11}\phi\right)\gamma^{\bar{a}}|\Omega\rangle &= 0\,, \end{aligned} \tag{5.1}$$

where we have used, as before, the decomposition (4.2). At first sight, equations (5.1) look quite complicated. However, their analysis can be facilitated by the following observations. Since they are linear in the flux, one can study the contributions of the $(2,1,1)$ and $(1,2,1)$ components separately. Furthermore, on physical grounds turning on background flux can only reduce the number of zero modes compared to the fluxless case.[25] However, the presence of flux can deform (some of) the surviving zero modes. Let us see what do the above considerations imply in our case. For vanishing flux the four zero modes (recall that for convenience we have suppressed the **4** index of ϕ, $\phi_{\bar{a}\bar{b}}$) were given by ϕ — harmonic and $\phi_{\bar{a}\bar{b}} = 0$. Hence, if it is possible to have $\phi_{\bar{a}\bar{b}} \neq 0$ for $G \neq 0$, then the $\phi_{\bar{a}\bar{b}}$ solution must be completely determined by the solution for ϕ, together with the flux. Otherwise the number of zero modes will increase by 3×4, which is the number of independent components of $\phi_{\bar{a}\bar{b}}$.

Now we are ready to start analyzing the system (5.1) in the presence of each of the two allowed types of flux. Let us begin with $(2,1,1)$ fluxes. In this case the equations become:

$$\begin{aligned}\partial_{[\bar{a}}\phi_{\bar{b}\bar{c}]} &= 0 \\ \partial_{\bar{a}}\phi + 4g^{\bar{b}c}\partial_c\phi_{\bar{b}\bar{a}} - 8G_{\bar{a}11bc}\phi^{bc} + 8G_{c11}{}^{c\bar{b}}\phi_{\bar{b}\bar{a}} &= 0\,. \end{aligned} \tag{5.2}$$

Acting on the second equation with $\nabla_{\bar{d}}$, together with antisymmetrizing w.r.t. $\bar{d}$ and $\bar{a}$, and adding to the result the action of $\nabla^{\bar{a}}$ on the first equation, we find:[26]

$$\Delta\phi_{\bar{d}\bar{a}} = 8\left(\nabla_{[\bar{d}|}G_{c11}{}^{c\bar{b}} + G_{c11}{}^{c\bar{b}}\nabla_{[\bar{d}|}\right)\phi_{\bar{b}|\bar{a}]} - 8\left(\partial_{[\bar{d}}G_{\bar{a}]11}{}^{\bar{b}\bar{c}} + G_{[\bar{a}|11}{}^{\bar{b}\bar{c}}\nabla_{|\bar{d}]}\right)\phi_{\bar{b}\bar{c}}\,. \tag{5.3}$$

The system (5.3) consists of three coupled equations for three unknown functions. Any solutions $\phi_{\bar{a}\bar{b}}$ are determined by the flux only. In other words, the $\phi_{\bar{a}\bar{b}}$ zero modes do not depend on ϕ. On the contrary, from the second equation in (5.2) one can determine ϕ in terms of the flux and the solutions for $\phi_{\bar{a}\bar{b}}$. Hence, the number of solutions is determined by $\phi_{\bar{a}\bar{b}}$ and therefore for generic flux it is larger than in the fluxless case. The only way to reconcile this with the observations in the paragraph below the system (5.1) is to assume that the solution is in fact $\phi_{\bar{a}\bar{b}} = 0$, which then implies that ϕ is harmonic. So the conclusion is that generic $(2,1,1)$ flux does not affect at all the zero modes.

[24] Not to be confused with the Minkowski gauge five-brane that may be a part of the background.

[25] The reason is that for nonzero flux there are additional supersymmetry constraints (for example, primitivity conditions for flux components). Satisfying them leads to smaller number of geometric moduli and hence also to smaller number of their superpartners, which are the fermionic moduli.

[26] Recall that $\Delta\phi_{\bar{a}\bar{b}} = 2\Delta_{\bar{\partial}}\,\phi_{\bar{a}\bar{b}} = -2\nabla^{\bar{c}}\nabla_{\bar{c}}\,\phi_{\bar{a}\bar{b}} - 4R_{\bar{a}}{}^{\bar{c}}{}_{\bar{b}}{}^{\bar{d}}\,\phi_{\bar{c}\bar{d}} = -2\nabla^{\bar{c}}\nabla_{\bar{c}}\,\phi_{\bar{a}\bar{b}}$, where the last equality is due to $R_{a\bar{b}c\bar{d}} = R_{a\bar{d}c\bar{b}}$ on a Kähler manifold.

Now let us turn to the $(1,2,1)$ type of flux. In this case the system (5.1) reduces to:

$$\begin{aligned} \partial_{[\bar a}\phi_{\bar b\bar c]} + 4G_{[\bar a\bar b}{}^{\bar d}{}_{|11|}\phi_{\bar c]\bar d} + 2G^{\bar d}{}_{\bar d[\bar a|11|}\phi_{\bar b\bar c]} &= 0 \\ \partial_{\bar a}\phi + 4g^{\bar b c}\partial_c\phi_{\bar b\bar a} + 2G_{\bar a\bar c}{}^{\bar c}{}_{11}\phi &= 0\,. \end{aligned} \tag{5.4}$$

Acting with $\nabla^{\bar a}$ on the second equation, we find:

$$\Delta\phi - 4G_{\bar a\bar c}{}^{\bar c}{}_{11}\partial^{\bar a}\phi - 4(\partial^{\bar a}G_{\bar a\bar c}{}^{\bar c}{}_{11})\phi = 0\,. \tag{5.5}$$

The second-order linear differential operator acting on ϕ in the above equation is clearly elliptic. Since we are on a compact manifold, its spectrum will be discrete and so nontrivial solutions for ϕ will exist only if one of the eigenvalues is zero. This is clearly the case for vanishing flux, when the operator reduces to the laplacian and hence ϕ is harmonic. But for nonzero flux (unless the flux is very particular) the eigenvalue will typically be shifted away from zero. We conclude that, generically, the only solution of (5.5) is $\phi = 0$. In other words, generic flux completely lifts the zero modes of the fluxless case.

To recapitulate, turning on generic flux of type $(1,2,1)$ lifts all zero modes. Again, there may be important exceptions for very special choices of flux. On the other hand, a background flux component of type $(2,1,1)$ does not affect the zero mode counting, similarly to the $(2,2,0)$ component. We should note though that the situation is reversed for anti-M5-brane instantons. Namely, all zero modes on their worldvolume are lifted by a generic $(2,1,1)$ flux, whereas the $(1,2,1)$ type of flux does not affect them. For more details see the appendix.

6. World-volume flux

Until now, our considerations of the Dirac equation on the worldvolume of an M5-brane instanton always neglected for simplicity (as in all existing literature) the self-dual three-form h. However, as we saw in section 4.2, h plays an important role in the resolution of the problem of reconciling five-brane instantons and gauged isometries. Hence, it is natural to ask how its presence affects the zero-mode counting of the previous sections. Unfortunately, taking into account both $h \neq 0$ (or equivalently, $H_{lmn} = (m^{-1})_l{}^p h_{mnp} \neq 0$) and nonvanishing background flux is too complicated to address in full generality. In the particular case of $(2,2,0)$ background though, the Dirac equation simplifies significantly and we will be able to analyze it in the presence of nonzero worldvolume flux. As a result, we will see that, whenever the topological constraints of section 4.2 (or the conditions in section 4.3) are satisfied, the presence of M5-brane instantons is allowed by the zero-mode counting even with $h \neq 0$.

6.1 Preliminaries

Since background fluxes of type $(2,2,0)$ (as those considered in section 4) do not contribute to the Dirac equation (4.3), for such backgrounds the latter simplifies to:

$$\gamma_A m^{AB}\nabla_B\theta = 0\,, \qquad m^{AB} = \delta^{AB} - 2h^A{}_{CD}h^{BCD}\,. \tag{6.1}$$

JHEP10(2006)071

To make further progress we will use the solution for h found in [41] (see also [42]):

$$h = c\Omega + \chi\,. \tag{6.2}$$

Here Ω is the CY $(3,0)$-form, χ is a primitive $(1,2)$-form and c is a constant, which for convenience we will absorb in the definition of Ω from now on. This is the most general form of the worldvolume flux for an M5 instanton wrapping a CY(3) in the absence of background flux.[27] Nevertheless, it is all we need since effectively the compatibility condition between the background flux and M5 instantons is that the pullback of the flux on the brane worldvolume be zero (see section 4.2). Substituting (6.2) in equation (6.1) and using the decomposition (4.2) of the world-volume fermions, we find:

$$\begin{aligned}\left(\partial_{\bar{a}}\phi + 4g^{b\bar{c}}\partial_b\phi_{\bar{c}\bar{a}} - 16\mu^{\bar{c}\bar{b}}\nabla_{\bar{b}}\phi_{\bar{c}\bar{a}} - 4\nu_{\bar{a}}{}^b\partial_b\phi\right)\gamma^{\bar{a}}|\Omega\rangle &= 0\\ \left(\partial_{[\bar{a}}\phi_{\bar{b}\bar{c}]} - 4\nu_{[\bar{a}}{}^d\partial_{|d|}\phi_{\bar{b}\bar{c}]}\right)\gamma^{\bar{a}\bar{b}\bar{c}}|\Omega\rangle &= 0\,,\end{aligned} \tag{6.3}$$

where for convenience we have introduced the combinations:[28]

$$\nu_{\bar{a}}{}^b = \chi_{\bar{a}c\bar{d}}\chi^{bc\bar{d}}\,, \qquad \mu_a{}^{\bar{b}} = \Omega_{acd}\chi^{\bar{b}cd}\,, \tag{6.4}$$

and used the relation $\mu^{\bar{a}\bar{b}} = \mu^{\bar{b}\bar{a}}$, whose origin will be recalled shortly. Note that, inverting the second relation above i.e. using $\chi_{a\bar{b}\bar{c}} = \mu_a{}^{\bar{d}}\bar{\Omega}_{\bar{d}\bar{b}\bar{c}}$, one can write $\nu_{\bar{a}}{}^b = \mu_c{}^{\bar{d}}\bar{\Omega}_{\bar{a}\bar{d}\bar{e}}\mu_g{}^{\bar{e}}\bar{\Omega}^{gbc}$.

In order to be able to solve equations (6.3), we will need one more result from [41]. Namely, the $(1,2)$-form χ, that determines the world-volume flux via (6.2), has to be of a very particular form. Let us briefly recall the reasons for that. The self-dual three-form h is determined by its equation of motion [35]:

$$m^{AB}\nabla_A h_{BCD} = 0\,. \tag{6.5}$$

The $(1,1)$ and $(0,2)$ components of the latter give respectively

$$\partial_{[a}\mu_{b]}{}^{\bar{c}} - 4c\mu_{[a}{}^{\bar{d}}\partial_{\bar{d}}\mu_{b]}{}^{\bar{c}} = 0\,, \tag{6.6}$$

which is exactly the Kodaira-Spencer equation [43] that describes finite deformations of the complex structure of the CY(3), and

$$\nabla^a\chi_{a\bar{b}\bar{c}} - 4\chi_{\bar{a}d\bar{e}}\chi^{fd\bar{e}}\nabla^{\bar{a}}\chi_{f\bar{b}\bar{c}} = 0\,, \tag{6.7}$$

whereas the $(2,0)$ component vanishes identically due to $\nabla\Omega = 0$. In addition, the primitivity condition $J \wedge \chi = 0$ leads to

$$\mu_{ab} = \mu_{ba}\,. \tag{6.8}$$

Equation (6.7) is a deformation of the gauge choice $\partial^\dagger\chi = 0$ in which the solution of (6.6) was found by Tian and Todorov [44]. This solution has the form

$$\chi = \sum_{n=1}^{\infty}\epsilon^n\chi^{(n)}\,, \tag{6.9}$$

[27] Nonzero background flux complicates significantly the field equation for h and the generic solution in that case is not known.

[28] Our definition of $\mu_a{}^{\bar{b}}$ differs by a factor of 1/2 from the one used in [41].

with ϵ being a small parameter, and satisfies (6.8) automatically.[29] As the precise form of the functions $\chi^{(n)}$ is not important for us, we will not write them down. It was argued in [41] that the Tian-Todorov solution can be deformed to a new one, still of the form (6.9), which satisfies the gauge condition (6.7) together with (6.6) and (6.8).[30] In view of this, we take the worldvolume flux parameters in the Dirac equation (6.3) to also be power series in ϵ:

$$\mu = \sum_{n=1}^{\infty} \epsilon^n \mu^{(n)}\,, \qquad \nu = \sum_{n=1}^{\infty} \epsilon^{2n} \nu^{(2n)}\,, \tag{6.10}$$

where the expansion of ν has only even powers because of (6.4). This implies that the solutions of (6.3) should also be power series:

$$\phi = \sum_{n=1}^{\infty} \epsilon^n \phi^{(n)}\,, \qquad \phi_{\bar{a}\bar{b}} = \sum_{n=1}^{\infty} \epsilon^n \phi^{(n)}_{\bar{a}\bar{b}}\,. \tag{6.11}$$

6.2 Solving the Dirac equation

Let us now start solving (6.3) order by order in ϵ. At first order we have the system:

$$\begin{aligned} \partial_{\bar{a}}\phi^{(1)} + 4g^{b\bar{c}}\partial_b\phi^{(1)}_{\bar{c}\bar{a}} &= 0 \\ \partial_{[\bar{a}}\phi^{(1)}_{\bar{b}\bar{c}]} &= 0\,, \end{aligned} \tag{6.12}$$

which implies that $\phi^{(1)}$ and $\phi^{(1)}_{\bar{a}\bar{b}}$ are harmonic. Using $h^{0,2}(CY(3)) = 0$, we find that $\phi^{(1)}_{\bar{a}\bar{b}} = 0$. Hence, at second order (6.3) gives again:

$$\begin{aligned} \partial_{\bar{a}}\phi^{(2)} + 4g^{b\bar{c}}\partial_b\phi^{(2)}_{\bar{c}\bar{a}} &= 0 \\ \partial_{[\bar{a}}\phi^{(2)}_{\bar{b}\bar{c}]} &= 0\,, \end{aligned} \tag{6.13}$$

since the only ϵ^2 term containing μ or ν would have been $16\mu^{(1)\bar{c}\bar{b}}\nabla_{\bar{b}}\phi^{(1)}_{\bar{c}\bar{a}}$. Therefore, $\phi^{(2)}$ and $\phi^{(2)}_{\bar{a}\bar{b}}$ are also harmonic and as a result $\phi^{(2)}_{\bar{a}\bar{b}} = 0$ too.

At order ϵ^3 we find a more complicated system:

$$\begin{aligned} \partial_{\bar{a}}\phi^{(3)} + 4g^{b\bar{c}}\partial_b\phi^{(3)}_{\bar{c}\bar{a}} - 4\nu^{(2)}{}_{\bar{a}}{}^{b}\partial_b\phi^{(1)} &= 0 \\ \partial_{[\bar{a}}\phi^{(3)}_{\bar{b}\bar{c}]} &= 0\,, \end{aligned} \tag{6.14}$$

where we have used the vanishing of $\phi^{(1)}_{\bar{a}\bar{b}}$ and $\phi^{(2)}_{\bar{a}\bar{b}}$. Note that these equations are of exactly the same form as (3.11) and (3.13) of [14] with $4\nu^{(2)}{}_{\bar{a}}{}^{b}\partial_b\phi^{(1)}$ playing the role of the inhomogeneous flux term. However, in our case things are even simpler as we have already found that $\phi^{(1)}$ is harmonic. Since a harmonic function on a compact space is necessarily constant, $\partial_b\phi^{(1)} = 0$. Therefore, we again find that $\phi^{(3)}$, $\phi^{(3)}_{\bar{a}\bar{b}}$ are harmonic and so $\phi^{(3)}_{\bar{a}\bar{b}} = 0$.

[29] This solution of the Kodaira-Spencer equation has also been considered in the context of the topological B-model in [45].

[30] We should note, that although [41] presents convincing arguments, it does not give a rigorous proof.

JHEP10(2006)071

It is easy to generalize the above considerations to any order, but before doing that, let us gain more familiarity with the equations involved by writing down the systems that result for two more iterations. At order ϵ^4 (6.3) gives:

$$\begin{aligned} \partial_{\bar{a}}\phi^{(4)} + 4g^{b\bar{c}}\partial_b\phi^{(4)}_{\bar{c}\bar{a}} - 16\mu^{(1)\bar{c}\bar{b}}\nabla_{\bar{b}}\phi^{(3)}_{\bar{c}\bar{a}} - 4\nu^{(2)}{}_{\bar{a}}{}^{b}\partial_b\phi^{(2)} &= 0 \\ \partial_{[\bar{a}}\phi^{(4)}_{\bar{b}\bar{c}]} &= 0\,, \end{aligned} \tag{6.15}$$

whereas at order ϵ^5:

$$\begin{aligned} \partial_{\bar{a}}\phi^{(5)} + 4g^{b\bar{c}}\partial_b\phi^{(5)}_{\bar{c}\bar{a}} - 16\sum_{k=1}^{2}\mu^{(k)\bar{c}\bar{b}}\nabla_{\bar{b}}\phi^{(5-k)}_{\bar{c}\bar{a}} - 4\sum_{k=1}^{2}\nu^{(2k)}{}_{\bar{a}}{}^{b}\partial_b\phi^{(5-2k)} &= 0 \\ \partial_{[\bar{a}}\phi^{(5)}_{\bar{b}\bar{c}]} - 4\nu^{(2)}{}_{[\bar{a}}{}^{d}\partial_{|d|}\phi^{(3)}_{\bar{b}\bar{c}]} &= 0\,. \end{aligned} \tag{6.16}$$

It is clear now that at order ϵ^n one has:

$$\begin{aligned} \partial_{\bar{a}}\phi^{(n)} + 4g^{b\bar{c}}\partial_b\phi^{(n)}_{\bar{c}\bar{a}} &= 4T^{(n)}_{\bar{a}} \\ \partial_{[\bar{a}}\phi^{(n)}_{\bar{b}\bar{c}]} &= S^{(n)}_{\bar{a}\bar{b}\bar{c}}\,, \end{aligned} \tag{6.17}$$

where for convenience we have introduced the notation

$$\begin{aligned} T^{(n)}_{\bar{a}} &= 4\sum_{k=1}^{n-3}\mu^{(k)\bar{c}\bar{b}}\nabla_{\bar{b}}\phi^{(n-k)}_{\bar{c}\bar{a}} + \sum_{k=1}^{[n/2]}\nu^{(2k)}{}_{\bar{a}}{}^{b}\partial_b\phi^{(n-2k)} \\ S^{(n)}_{\bar{a}\bar{b}\bar{c}} &= 4\sum_{k=1}^{[(n-3)/2]}\nu^{(2k)}{}_{[\bar{a}}{}^{d}\partial_{|d|}\phi^{(n-2k)}_{\bar{b}\bar{c}]}\,, \end{aligned} \tag{6.18}$$

and the upper limits in the sums take into account that $\phi^{(1)}_{\bar{a}\bar{b}}, \phi^{(2)}_{\bar{a}\bar{b}} = 0$. Obviously $T^{(n)}_{\bar{a}}$ and $S^{(n)}_{\bar{a}\bar{b}\bar{c}}$ depend only on $\phi^{(k)}$ and $\phi^{(k)}_{\bar{a}\bar{b}}$ with $k < n$, which at the previous stages have been shown to be harmonic. The latter fact implies that $\phi^{(k)}_{\bar{a}\bar{b}} = 0$ and $\partial_b\phi^{(k)} = 0$, which leads to

$$T^{(n)}_{\bar{a}} = 0\,, \qquad S^{(n)}_{\bar{a}\bar{b}\bar{c}} = 0\,. \tag{6.19}$$

Hence $\phi^{(n)}$ and $\phi^{(n)}_{\bar{a}\bar{b}}$ are also harmonic.

To recapitulate, the solution of (6.3) is given by $\phi_{\bar{a}\bar{b}} = 0$ and ϕ — harmonic. Since $h^{0,0}(CY(3)) = 1$, we find a single zero mode. Taking into account the **4** index that we have suppressed for convenience, this means that there are four zero modes just as in the case without world-volume flux.

7. Discussion

In this work we considered the interplay between flux-induced gauging of isometries and M5-brane instantons in five-dimensional heterotic M-theory. We showed that the reconciliation of the above two competing effects is due to the enforcement of the Gauss' law on the instanton worldvolume. It occurs for CY 3-folds that satisfy certain topological constraints.

We explained that these constraints are significantly eased by considering compactifications with nonstandard embedding. In addition, we investigated in detail the Dirac equation for the M5 worldvolume fermions in the presence of all possible types of supersymmetric background flux. It turned out that backgrounds of type $(2,2,0)$ and $(2,1,1)$ do not change the zero-mode counting of the fluxless case, whereas flux of type $(1,2,1)$ lifts all zero modes. (For anti-M5 instantons the roles of the $(2,1,1)$ and $(1,2,1)$ fluxes are reversed.) We also managed, for first time, to solve the Dirac equation with nonvanishing worldvolume flux, although under restricted conditions.

In heterotic M-theory there is always background flux, as we recalled in the introduction. So it was indeed pressing to show the consistency of the gauged supergravity description when non-perturbative effects are taken into account. However, clearly one can turn on background flux in M-theory compactifications to five or four dimensions as well. While the four-dimensional case is important mostly in the context of moduli stabilization, the five-dimensional one is relevant also for the domain-wall/QFT correspondence [46] and supersymmetric realizations of the Randall-Sundrum scenario [47]. With the latter motivation in mind, the work [48] studied M-theory compactifications to 5d with background flux and, in particular, derived the flux-induced gauging in the effective supergravity description[31], similarly to the results of [50] for type II strings. It turns out that again the isometry of the universal hypermultiplet moduli space, that is given by constant shifts of the axionic scalar σ, is gauged by the flux.[32] Hence the considerations of the present paper apply, pretty much literally, to this case as well. Finally, it is certainly of interest to also study, in the same vein as here, the zero mode counting on the worldvolume of membranes in M-theory flux compactifications to 4d as M2 instantons could contribute to the superpotential of the low-energy effective theory.

Acknowledgments

We would like to thank A. Kashani-Poor for valuable correspondence and A. Lukas and D. Waldram for discussions. We are also grateful to the referee for insightful comments that helped improve the quality of the paper. The work of L.A. is supported by the EC Marie Curie Research Training Network MRTN-CT-2004-512194 *Superstrings*. K.Z. is supported by a PPARC grant "Gauge theory, String Theory and Twistor Space Techniques".

A. Zero-mode counting for anti-M5 instantons

In the main text we considered M5-branes and, in order to construct an explicit representation of the fermion states on their worldvolume, we defined the Clifford vacuum by $\gamma^a|\Omega\rangle = 0$. Then the states are obtained by acting on $|\Omega\rangle$ with the creation operators $\gamma^{\bar{a}}$. To represent the fermion states on an anti-M5-brane worldvolume one can define another

[31] For more work on solutions in this theory see e.g. [49].

[32] For classification of all possible (irrespective of background flux) gaugings of the moduli space of the universal hypermultiplet see [51].

JHEP10(2006)071

Clifford vacuum $|\Omega'\rangle$ by

$$\gamma^{\bar{a}}|\Omega'\rangle = 0 \ . \tag{A.1}$$

Now the creation operators are γ^a and so the decomposition of the worldvolume spinor θ' is:

$$\theta' = \phi'|\Omega'\rangle + \phi'_{ab}\gamma^{ab}|\Omega'\rangle \ . \tag{A.2}$$

This is in accord with the realization of anti-brane worldvolume states in string theory as complex conjugates of the corresponding brane states. Therefore, it is immediately obvious that anti-M5 instantons couple to the fluxes of type $(2,1,1)$ and $(1,2,1)$ in an opposite way compared to the M5 instantons. Hence it follows from the results of section 5 that the $(2,1,1)$ flux lifts all of their zero-modes, whereas the $(1,2,1)$ flux does not affect them.

It is worth noting that, unlike the case of anti-D-branes, for anti-M5-branes there is an alternative representation of the fermionic states. This is due to the fact that their world-volume spinor θ' has definite chirality, which is correlated with the self-duality properties of the world-volume three-form h. More precisely, θ' is anti-chiral and so can be built as an expansion in terms of odd number of creation operators acting on the original vacuum $|\Omega\rangle$:

$$\theta' = \phi_{\bar{a}}\gamma^{\bar{a}}|\Omega\rangle + \phi_{\bar{a}\bar{b}\bar{c}}\gamma^{\bar{a}\bar{b}\bar{c}}|\Omega\rangle. \tag{A.3}$$

Clearly, if the two representations (A.2) and (A.3) are to describe the same physics, they have to be equivalent. And indeed they are, since one can write an explicit mapping between them:

$$|\Omega'\rangle = \bar{\Omega}_{\bar{a}\bar{b}\bar{c}}\gamma^{\bar{a}\bar{b}\bar{c}}|\Omega\rangle \qquad \text{and} \qquad \phi' = \Omega^{\bar{a}\bar{b}\bar{c}}\phi_{\bar{a}\bar{b}\bar{c}} \quad , \quad \phi'_{ab} = \Omega_{ab}{}^{\bar{c}}\phi_{\bar{c}} \ , \tag{A.4}$$

which is essentially the statement of Serre duality for the Calabi-Yau 3-fold.

References

[1] K. Dasgupta, G. Rajesh and S. Sethi, *M-theory, orientifolds and G-flux*, *JHEP* **08** (1999) 023 [hep-th/9908088].

[2] S.B. Giddings, S. Kachru and J. Polchinski, *Hierarchies from fluxes in string compactifications*, *Phys. Rev.* **D 66** (2002) 106006 [hep-th/0105097].

[3] M. Graña, *Flux compactifications in string theory: a comprehensive review*, *Phys. Rept.* **423** (2006) 91 [hep-th/0509003].

[4] S. Kachru, R. Kallosh, A. Linde and S.P. Trivedi, *de Sitter vacua in string theory*, *Phys. Rev.* **D 68** (2003) 046005 [hep-th/0301240].

[5] A.-K. Kashani-Poor and A. Tomasiello, *A stringy test of flux-induced isometry gauging*, *Nucl. Phys.* **B 728** (2005) 135 [hep-th/0505208].

[6] P. Hořava and E. Witten, *Heterotic and type-I string dynamics from eleven dimensions*, *Nucl. Phys.* **B 460** (1996) 506 [hep-th/9510209]; *Eleven-dimensional supergravity on a manifold with boundary*, *Nucl. Phys.* **B 475** (1996) 94 [hep-th/9603142].

[7] E. Witten, *Strong coupling expansion of Calabi-Yau compactification*, *Nucl. Phys.* **B 471** (1996) 135 [hep-th/9602070].

JHEP10(2006)071

[8] T. Banks and M. Dine, *Couplings and scales in strongly coupled heterotic string theory*, *Nucl. Phys.* **B 479** (1996) 173 [hep-th/9605136].

[9] A. Lukas, B.A. Ovrut and D. Waldram, *Cosmological solutions of Hořava-Witten theory*, *Phys. Rev.* **D 60** (1999) 086001 [hep-th/9806022];
H.S. Reall, *Open and closed cosmological solutions of Hořava-Witten theory*, *Phys. Rev.* **D 59** (1999) 103506 [hep-th/9809195];
U. Ellwanger, *Cosmological evolution in compactified Hořava-Witten theory induced by matter on the branes*, *Eur. Phys. J.* **C 25** (2002) 157 [hep-th/0001126];
M. Brandle, A. Lukas and B.A. Ovrut, *Heterotic M-theory cosmology in four and five dimensions*, *Phys. Rev.* **D 63** (2001) 026003 [hep-th/0003256];
C. van de Bruck, M. Dorca, R.H. Brandenberger and A. Lukas, *Cosmological perturbations in brane-world theories: formalism*, *Phys. Rev.* **D 62** (2000) 123515 [hep-th/0005032];
A. Kehagias and K. Tamvakis, *A note on brane cosmology*, *Phys. Lett.* **B 515** (2001) 155 [hep-ph/0104195];
R. Arnowitt, J. Dent and B. Dutta, *Five dimensional cosmology in Hořava-Witten M-theory*, *Phys. Rev.* **D 70** (2004) 126001 [hep-th/0405050].

[10] A. Lukas, B.A. Ovrut, K.S. Stelle and D. Waldram, *The Universe as a domain wall*, *Phys. Rev.* **D 59** (1999) 086001 [hep-th/9803235].

[11] A. Lukas, B.A. Ovrut, K.S. Stelle and D. Waldram, *Heterotic M-theory in five dimensions*, *Nucl. Phys.* **B 552** (1999) 246 [hep-th/9806051].

[12] G. Curio and A. Krause, *S-track stabilization of heterotic de Sitter vacua*, hep-th/0606243.

[13] K. Becker and M. Becker, *Instanton action for type-II hypermultiplets*, *Nucl. Phys.* **B 551** (1999) 102 [hep-th/9901126].

[14] R. Kallosh, A.-K. Kashani-Poor and A. Tomasiello, *Counting fermionic zero modes on M5 with fluxes*, *JHEP* **06** (2005) 069 [hep-th/0503138].

[15] N. Saulina, *Topological constraints on stabilized flux vacua*, *Nucl. Phys.* **B 720** (2005) 203 [hep-th/0503125].

[16] E. Bergshoeff, R. Kallosh, A.-K. Kashani-Poor, D. Sorokin and A. Tomasiello, *An index for the dirac operator on d3 branes with background fluxes*, *JHEP* **10** (2005) 102 [hep-th/0507069].

[17] L. Görlich, S. Kachru, P.K. Tripathy and S.P. Trivedi, *Gaugino condensation and nonperturbative superpotentials in flux compactifications*, *JHEP* **12** (2004) 074 [hep-th/0407130].

[18] K. Becker, M. Becker and A. Strominger, *Five-branes, membranes and nonperturbative string theory*, *Nucl. Phys.* **B 456** (1995) 130 [hep-th/9507158];
J.A. Harvey and G.W. Moore, *Superpotentials and membrane instantons*, hep-th/9907026.

[19] E. Witten, *Non-perturbative superpotentials in string theory*, *Nucl. Phys.* **B 474** (1996) 343 [hep-th/9604030].

[20] C. Vafa, *Evidence for F-theory*, *Nucl. Phys.* **B 469** (1996) 403 [hep-th/9602022].

[21] R. Kallosh and D. Sorokin, *Dirac action on M5 and M2 branes with bulk fluxes*, *JHEP* **05** (2005) 005 [hep-th/0501081].

[22] S. Ferrara and S. Sabharwal, *Dimensional reduction of type-II superstrings*, *Class. and Quant. Grav.* **6** (1989) L77.

[23] X.G. Wen and E. Witten, *World sheet instantons and the Peccei-Quinn symmetry*, *Phys. Lett.* **B 166** (1986) 397.

[24] A. Strominger, *Loop corrections to the universal hypermultiplet*, *Phys. Lett.* **B 421** (1998) 139 [`hep-th/9706195`].

[25] I. Antoniadis, R. Minasian, S. Theisen and P. Vanhove, *String loop corrections to the universal hypermultiplet*, *Class. and Quant. Grav.* **20** (2003) 5079 [`hep-th/0307268`].

[26] L. Anguelova, M. Roček and S. Vandoren, *Quantum corrections to the universal hypermultiplet and superspace*, *Phys. Rev.* **D 70** (2004) 066001 [`hep-th/0402132`].

[27] B. de Wit and A. Van Proeyen, *Symmetries of dual quaternionic manifolds*, *Phys. Lett.* **B 252** (1990) 221;
B. de Wit, F. Vanderseypen and A. Van Proeyen, *Symmetry structure of special geometries*, *Nucl. Phys.* **B 400** (1993) 463 [`hep-th/9210068`].

[28] M. Davidse, U. Theis and S. Vandoren, *Fivebrane instanton corrections to the universal hypermultiplet*, *Nucl. Phys.* **B 697** (2004) 48 [`hep-th/0404147`];
M. Davidse, F. Saueressig, U. Theis and S. Vandoren, *Membrane instantons and de Sitter vacua*, *JHEP* **09** (2005) 065 [`hep-th/0506097`].

[29] M. Gutperle and M. Spalinski, *Supergravity instantons and the universal hypermultiplet*, *JHEP* **06** (2000) 037 [`hep-th/0005068`].

[30] Z. Lalak, A. Lukas and B.A. Ovrut, *Soliton solutions of M-theory on an orbifold*, *Phys. Lett.* **B 425** (1998) 59 [`hep-th/9709214`].

[31] E. Lima, B.A. Ovrut, J. Park and R. Reinbacher, *Non-perturbative superpotential from membrane instantons in heterotic M-theory*, *Nucl. Phys.* **B 614** (2001) 117 [`hep-th/0101049`].

[32] D. Lüst, S. Reffert, W. Schulgin and P.K. Tripathy, *Fermion zero modes in the presence of fluxes and a non-perturbative superpotential*, *JHEP* **08** (2006) 071 [`hep-th/0509082`].

[33] I.A. Bandos et al., *Covariant action for the super-five-brane of M-theory*, *Phys. Rev. Lett.* **78** (1997) 4332 [`hep-th/9701149`]; *On the equivalence of different formulations of the M-theory five-brane*, *Phys. Lett.* **B 408** (1997) 135 [`hep-th/9703127`].

[34] P.S. Howe and E. Sezgin, $D = 11$, $p = 5$, *Phys. Lett.* **B 394** (1997) 62 [`hep-th/9611008`].

[35] P.S. Howe, E. Sezgin and P.C. West, *The six-dimensional self-dual tensor*, *Phys. Lett.* **B 400** (1997) 255 [`hep-th/9702111`].

[36] E. Witten, *New issues in manifolds of* SU(3) *holonomy*, *Nucl. Phys.* **B 268** (1986) 79;
L. Witten and E. Witten, *Large radius expansion of superstring compactifications*, *Nucl. Phys.* **B 281** (1987) 109.

[37] K. Benakli, *Scales and cosmological applications of M-theory*, *Phys. Lett.* **B 447** (1999) 51 [`hep-th/9805181`];
Z. Lalak, S. Pokorski and S. Thomas, *Beyond the standard embedding in M-theory on* $S^1/\mathbb{Z}_2$, *Nucl. Phys.* **B 549** (1999) 63 [`hep-ph/9807503`];
R. Donagi, A. Lukas, B.A. Ovrut and D. Waldram, *Non-perturbative vacua and particle physics in M-theory*, *JHEP* **05** (1999) 018 [`hep-th/9811168`];
A. Lukas, B.A. Ovrut and D. Waldram, *Five-branes and supersymmetry breaking in M-theory*, *JHEP* **04** (1999) 009 [`hep-th/9901017`];

JHEP10(2006)071

Z. Lalak and S. Thomas, *Scales of gaugino condensation and supersymmetry breaking in nonstandard M-theory embeddings*, *Nucl. Phys.* **B 575** (2000) 151 [hep-th/9908147].

[38] A. Lukas, B.A. Ovrut and D. Waldram, *Non-standard embedding and five-branes in heterotic M-theory*, *Phys. Rev.* **D 59** (1999) 106005 [hep-th/9808101].

[39] G. Curio and A. Krause, *Four-flux and warped heterotic M-theory compactifications*, *Nucl. Phys.* **B 602** (2001) 172 [hep-th/0012152];
L. Anguelova and D. Vaman, R^4 *corrections to heterotic M-theory*, *Nucl. Phys.* **B 733** (2006) 132 [hep-th/0506191].

[40] A. Strominger, *Heterotic solitons*, *Nucl. Phys.* **B 343** (1990) 167.

[41] M. Marino, R. Minasian, G.W. Moore and A. Strominger, *Nonlinear instantons from supersymmetric p-branes*, *JHEP* **01** (2000) 005 [hep-th/9911206].

[42] L. Bao, M. Cederwall and B.E.W. Nilsson, *A note on topological m5-branes and string-fivebrane duality*, hep-th/0603120.

[43] K. Kodaira and D. C. Spencer, *On deformations of complex analytic structures, I-II, III*, *Annals of Math.* **67** (1958) 328; *Annals of Math* **71** (1960) 43.

[44] G. Tian, *Smoothness of the universal deformation space of compact Calabi-Yau manifolds and its Petersson-Weil metric*, in *Mathematical aspects of string theory*, S. T. Yau, ed., World Scientific, 1987;
A. Todorov, *The Weil-Petersson geometry of the moduli space of* $SU(n \geq 3)$ *(Calabi-Yau) manifolds, I*, *Commun. Math. Phys.* **126** (1989) 325.

[45] M. Bershadsky, S. Cecotti, H. Ooguri and C. Vafa, *Kodaira-Spencer theory of gravity and exact results for quantum string amplitudes*, *Commun. Math. Phys.* **165** (1994) 311 [hep-th/9309140].

[46] H.J. Boonstra, K. Skenderis and P.K. Townsend, *The domain wall/QFT correspondence*, *JHEP* **01** (1999) 003 [hep-th/9807137].

[47] L. Randall and R. Sundrum, *A large mass hierarchy from a small extra dimension*, *Phys. Rev. Lett.* **83** (1999) 3370 [hep-ph/9905221]; *An alternative to compactification*, *Phys. Rev. Lett.* **83** (1999) 4690 [hep-th/9906064].

[48] K. Behrndt and S. Gukov, *Domain walls and superpotentials from M-theory on Calabi-Yau three-folds*, *Nucl. Phys.* **B 580** (2000) 225 [hep-th/0001082].

[49] M. Gutperle and W.A. Sabra, *A supersymmetric solution in* $N = 2$ *gauged supergravity with the universal hypermultiplet*, *Phys. Lett.* **B 511** (2001) 311 [hep-th/0104044].

[50] J. Louis and A. Micu, *Type II theories compactified on Calabi-Yau threefolds in the presence of background fluxes*, *Nucl. Phys.* **B 635** (2002) 395 [hep-th/0202168].

[51] K. Behrndt and M. Cvetič, *Gauging of* $N = 2$ *supergravity hypermultiplet and novel renormalization group flows*, *Nucl. Phys.* **B 609** (2001) 183 [hep-th/0101007].

JHEP10(2006)071